

## **Principles and Practical Applications of the Optimal Values Management and Programming**

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**Abstract:** In this paper presented are the methods for solving optimization problems, when there is a need to find the optimal value. This value may be minimum or maximum range. For example: the required profit and expenses of a company or institution, transport & costs minimization, computer network management, economic management of financial requirements. Also, given is an adequate method to optimize the function (objective function). The simplex method, Pivot method, and another approximation method, are the model that can help solving the question of the max and min problem. Here, presented are the program codes and the same implemented into C ++ and codes MATLAB. By applying these methods of optimization, we can have a direct approach and chance to minimize or maximize the objective function: which is the key point in creating certain optimum value. An optimization method has a practical use; this practical use is demonstrated through certain examples.

**Keywords:** *Optimal, values, model, minimum, maximum, objective function.*

### **Introduction**

The objectives function is dependent from subject variables. The subject is contained from mathematical inequalities, under certain conditions. The main point is how to manage the values that are given as mathematical inequalities and how to extract them to the adequate form. The subject is also an integral part within the decision variables and additional variables (Marsden & Tromba, 2003).

Manufacturers seek to use limited amounts of goods to achieve a greater profit. Managers of companies seek to align the workforce and business processes in order to complete tasks at a shorter time span. The purpose of these problems is optimization, utility maximization or minimization of costs to limit the data that is selected with linear programming. The area of application of linear programming is extensive: in manufacturing, transportation and distribution, marketing, telecommunications, financial organization and planning, alignment of employees *etc.* The formulation (modeling) of real life problem as a linear programming problem requires teamwork of experts from many fields (Marsden & Tromba, 2003).

### **The procedures of the Graph construction of linear inequalities with two and more variables**

The graph of linear inequalities with two variables includes the visual appearance of the points that fulfill the request of inequality (the area of feasibility). The graph would be half plane. There are 4 types to various publications inequalities with two variables:

$$Ax + By < C, Ax + By \leq C, Ax + By > C, Ax + By \geq C \quad (1)$$

Where A, B, C are constants. For inequalities graph:

$$Ax + By < C, Ax + By \leq C, Ax + By > C, Ax + By \geq C. \quad (2)$$

1. With equality  $Ax + By = C$  (border management) as defined points in the coordinates  $(T_1(0, y)$  and  $T_2(x, 0)$ ) and withdraw, straight line through these points. Thus, direction drawn will be with broken lines, when  $<$  or  $>$  and full line when  $\leq$  or  $\geq$ .
2. Solving the system is half-plane direction on one side of the border. To determine the side of the half plane, has been used a probation point  $T(a, b)$  which is not in the direction  $Ax + By = C$ . More often the point of the origin is used  $T(0, 0)$ . If this point doesn't accomplish the conditions for example  $Ax + By \geq C$ , so the graph of half plan is in another side, otherwise is next side and so on (Ben-Tal & Nemirovski, 2002).

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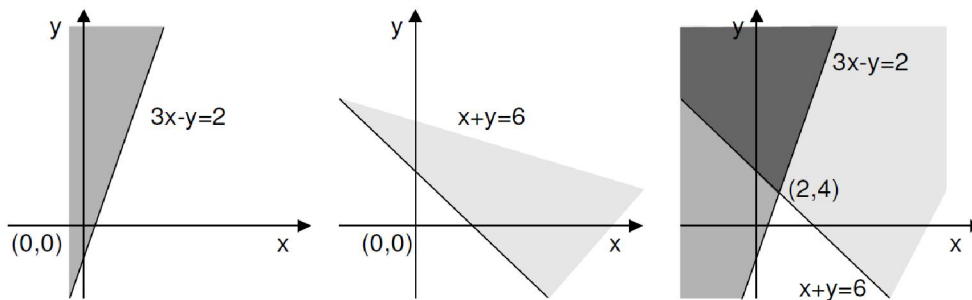
**Determination of the feasibility zone**

The following conditions will introduce the possibility of determining the feasibility of the system areas of the following inequalities:

$$\begin{aligned} A_1x + B_1y &\leq C_1 \\ A_2x + B_2y &\geq C_2 \end{aligned} \quad (3)$$

(for example,  $A_1=3, B_1=-1, C_1=2, A_2=1, B_2=1, C_2=6$ ) The feasibility area is the zone where the graphs overlap (and inequalities have common solution for the system).

Possible solutions are defined within the given space constraints. In fact, the optimal solution lies on the roofs. Exceptions are when border management complies with the function of the goal, and then two roofs and all points in the direction between the two roofs are the same optimal solution. The roofs of certain feasibility zone located within this area, cutting between two border directions (P. Kall and S. W. Wallace,1994).



**Figure 1.** Feasibility area is the zone where the graphs are overlapped

**Maximum Value, linear program, Additional variables and initial table**

Standard maximum problem is particular type of linear program which is easy to analyze. This is the linear program with which we wish to maximize the objective function F.

$$F = c_1x_1 + c_2x_2 + \dots + c_nx_n \quad (4)$$

The limitations are in the form of::

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{1n}x_n &\leq b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{2n}x_n &\leq b_2 \\ \vdots & \\ a_{m1}x_1 + a_{m2}x_2 + a_{mn}x_n &\leq b_m \\ x_1, x_2, \dots, x_n &\geq 0 \end{aligned}$$

Where  $b_j \geq 0$  for  $j = 1, 2, \dots, m$  In general, the problem of the maximum standard is linear program that meets the following conditions[1][2]:

- Objective function F must be maximized.
- All the limitations are as a types of the  $\leq$ .
- All the decisions variables  $x_i$  must be with positive values ( $x_i \geq 0$ ).
- All the values in the right side of the inequalities can be positive ( $b_j \geq 0$ ). [9][10]

The first step of the simplex method for solving the problem of the maximum standardsis to formulate restrictions and the objective function as inequalities. This procedure enables the implementation to solving the linear program (Saul I. Gass, 2003).

$$\begin{aligned} 3x_1 + 2x_2 - x_3 &\leq 4 \\ x_1 - 2x_2 &\leq 3 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

Added are additional variables  $u_1$  and  $u_2$  variable, so that the system obtained the following inequalities:

$$\begin{aligned} 3x_1 + 2x_2 - x_3 + u_1 &= 4 \\ x_1 - 2x_2 + u_2 &= 3 \\ x_1, x_2, x_3, u_1, u_2 &\geq 0 \end{aligned}$$

For example,

the function  $F = 2x_1 + 4x_2 - 7x_3$  can be written in the form  $-2x_1 - 4x_2 + 7x_3 + F = 0$ .

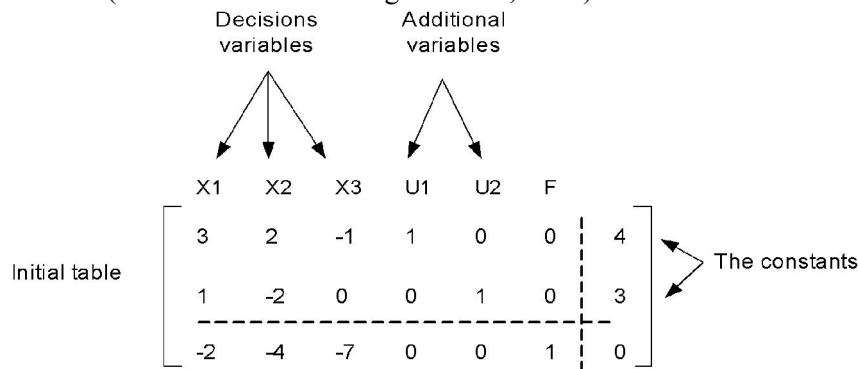
**Table 1.** The function objectives, and the conditions to maximize F.

<b>Function to be Maximized:</b>	$F = 2x_1 + 4x_2 - 7x_3$
<b>The subject and conditions:</b>	$3x_1 + 2x_2 - x_3 \leq 4$
	$x_1 - 2x_2 \leq 3$
	$x_1, x_2, x_3, u_1, u_2 \geq 0$

In the form of equations:

$$\begin{aligned} 3x_1 + 2x_2 - x_3 + u_1 &= 4 \\ x_1 - 2x_2 + u_2 &= 3 \\ -2x_1 - 4x_2 + 7x_3 + F &= 0 \\ x_1, x_2, x_3, u_1, u_2 &\geq 0 \end{aligned}$$

The matrix coefficients of the system of inequalities is called initial simplex table. In this example, the initial table is as follows (Steven Benson and Jorge J. More, 2001):



**Figure 2.** Decisions variables and additional variables

The system is presented in the form of linear inequalities system,

$$\begin{aligned} 3x_1 + 2x_2 - x_3 + u_1 &= 4 \\ x_1 - 2x_2 + u_2 &= 3 \end{aligned} \tag{5}$$

Where  $x_1, x_2, x_3, u_1$  and  $u_2$  are positive. There is infinite number of solutions to this system because it can be solved for any two variables. However, the solution is realized only if all members are non-negative.

Then all  $x_1, x_2, x_3$  are arbitrarily chosen  $u_1$  and  $u_2$ , and the linear system possesses a unique solution  $(x_1, x_2, x_3, u_1, u_2)$ .

When the linear system, written in the form  $u_1$  and  $u_2$ , called basic variables, and  $x_1, x_2$  and  $x_3$  They called variables, without basis (parametric). When the values of all variables without foundation are set to 0, the solution that is obtained is called basic solution (S. Lang, 1987).

In this example, the basic solution  $x_1 = 0, x_2 = 0, x_3 = 0, u_1 = 4$  and  $u_2 = 3$ . Since all solutions are nonnegative variables, hence this feasible solution is called basic feasible solution (S. Lang, 1987).

The given linear system can have many basic solutions, but not all solutions must be feasible. For example, if the linear system is:

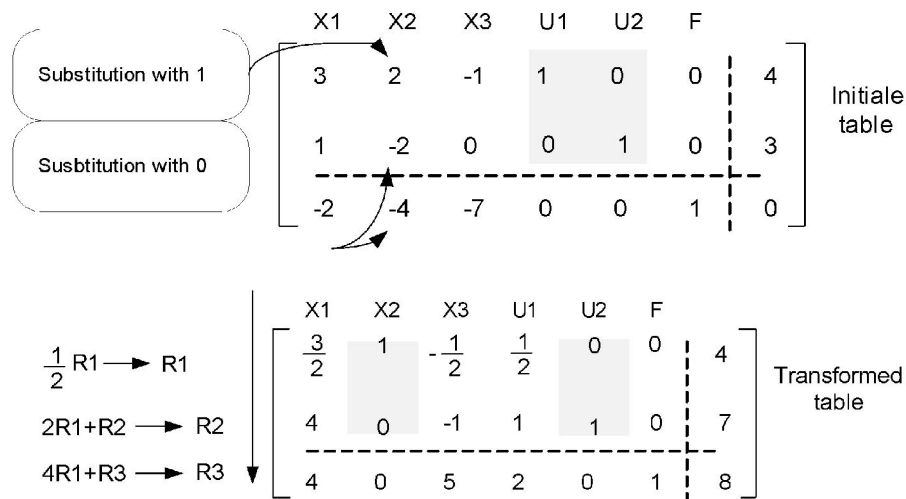
$$\begin{aligned} x_3 &= -4 + 3x_1 + 2x_2 + u_1 \\ u_2 &= 3 - x_1 + 2x_2 \end{aligned}$$

In this case  $u_2$  and  $x_3$  are variables without basis, and  $x_1, x_2$  and  $u_1$  (variable without base). They are equal to zero. The relevant resolution will be  $x_1 = 0, x_2 = 0, x_3 = -4, u_1 = 0$  and  $u_2 = 3$ , but in this case the basic solution is not feasible because of the variable is  $(x_3 = -4)$ .

**The model through Pivot method, and optimal value. Software management, C++ and MATLAB**

It is evident that the initial table of maximum standard linear program responds to the basic achievable solution in which all of the decision variables are 0. Since the objective function is equal to zero at the origin, is less likely that this may be the optimal solution. The simplex method starts from the origin to

proceed in the way of each peak (within the area of feasibility) until we find the roof that answers for the optimal solution. Pivot of the coefficient which is located at the position (1,2), ie, replaces the transformation coefficient 2 (in position (1,2)) with coefficient 1, and all others coefficients in the column become zero (C. H. Papadimitriou and K. Steiglitz, 1998).



**Figure 3.** The Pivot method and optimal value  
The new table responds to the linear system:

$$\begin{aligned} \frac{3}{2}x_1 + x_2 - \frac{1}{2}x_3 + \frac{1}{2}u_1 &= 4 \\ 4x_1 - x_3 + u_1 + u_2 &= 7 \\ 4x_1 + 5x_3 + 2u_1 + F &= 8 \end{aligned}$$

Which we can write in the following form:

$$\begin{aligned} x_2 &= 2 - \frac{3}{2}x_1 + \frac{1}{2}x_3 - \frac{1}{2}u_1 \\ u_2 &= 7 - 4x_1 + x_3 - u_1 \\ F &= 8 - 4x_1 - 5x_3 - 2u_1 \end{aligned}$$

for  $x_1 = 0, x_2 = 2, x_3 = 0, u_1 = 0$  and  $u_2 = 7$ . The roof includes the relevant variables  $i$ , and the coordinates is (0,2,0).

The value of the objective function is defined by solutions replacement.

$$F = 8 - 4 \cdot 0 + 5 \cdot 0 + 2 \cdot 0 = 8$$

In the next step is given the program how to find the indicators in the respective row. The indicators are in the last row. The requirement is to find the smaller indicator, for the “pivot” column,

**Program in the C++**

```
#include <iostream>
#include <iomanip>
using namespace std;
int main()
{
    constint m = 3, n = 7;
    intconst A[m][n] = { { 3,2,-1,1,0,0,4 },
        { 1,-4,-7,0,0,1,0 },
        { -2,-4,-7,0,0,1,0 } };
    int i, j, k = 0;
    int small ;
    cout<< "\n This row represent the row of the indicators: \n"
        <<endl;
    for (i = 2;i<m;i++)
    {
        small = A[i][0];
        for (j = 0;j<n;j++)
```

```
{
    cout<<setw(4)
    << A[i][j];
if (A[i][j] < small)
    small = A[i][j];
}
    cout<<endl;
}

cout<< "\nThe smaller indicator is ind="
    <<small
    << "\n\n";
cout<< "\nThis indicator represent the Pivot column"
    << "\n\n";
cin.get();cin.get();
return 0;
}
```

The next MATLAB code calculates or computes the profit function. This objective function can be realized by uses the function: `linprog (f,A,b) % optimize`. Where the f-is objective function, A-is the matrix and b-is a vector.

#### Matlab code #1:

```
>> f = [-143 -60]
>> A = [120 210; 110 30; 1 1; -1 0; 0 -1]
>> b = [15000; 4000; 75; 0; 0]
The optimization toolbox has the command linprog:
>>linprog(f,A,b) % optimize
>> -f*ans % compute profit etc.
```

#### Conclusion

The paper gives us an interesting access to the simplex algorithm and Pivot method for linear programming of the maximum standard type and the same must meet the following requirements:

- To enables the initial table for the given problem of the maximum - standard.
- To divide each constant on the right side, with all members of the relevant positive pivot column.
- To match the "pivot" with the transformation table.
- Here is needed to repeat steps 1-3 until all the indicators take positive values.

In the paper are given adequate models and methods for optimization of the objective function, which is followed by the subject, namely the conditions are necessary for meeting the demand for certain minimum value. Also in the paper is provided program codes implemented in C++ programming language, and also is used the codes in MATLAB.

Also, on this paper, the provided models and methods for minimizing or maximization of the objective function. This objective function is dependent from the mathematical model and form the conditions listed in the subject. Subject contains mathematical inequalities, under certain conditions.

Note: If all members on the most negative indicator were negative, the smallest ratio cannot be found, thus in this case the linear program doesn't have the optimal solution.

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